UDC 539. 3

ON THE THEORY OF DIFFRACTION OF LINEAR VISCO-ELASTIC WAVES

PMM Vol. 42, № 1, 1978. pp. 159-167

I. G. FILIPPOV

(Moscow)

(Received March 11, 1977)

Two-dimensional problems of diffraction of visco-elastic waves on rigid contours of arbitrary form especially on rectilinear cuts, are studied in the linear approximation. Solutions are obtained using a generalization of the Volterra method.

The problems in question were studied in [1–7] and solved either using the Volterra method, or some other methods.

1. Formulation of the general problem of diffraction of visco-elastic waves and its solution. We shall assume that the medium is isotropic and instantaneously elastic and, that the kernels of the visco-elastic operators are arbitrary. We shall express the relations between the stress and deformation tensors in the form of the integral Boltzmann relations

$$\sigma_{jj} = L(\varepsilon) + 2M(\varepsilon_{jj}) \quad (j = x, y), \quad \sigma_{xy} = M(\varepsilon_{xy}) \tag{1.1}$$

where $L(\zeta)$ and $M(\zeta)$ are linear integral operators, while $h_j(\alpha)$ and γ_{mj} are continuous and discrete functions of the relaxation times

$$L(\zeta) = \lambda\left[\zeta(t) - \int_0^t f_1(t-\xi)\,\zeta(\xi)\,d\xi\right]$$

$$M(\zeta) = \mu\left[\zeta(t) - \int_0^t f_2(t-\xi)\,\zeta(\xi)\,d\xi\right]$$

$$f_j(t) = \int_0^\infty h_j(\alpha)\exp\left(-\frac{t}{\alpha}\right)d\alpha + \sum_{m=1}^{m_0}\frac{\gamma_{mj}}{\tau_m}\exp\left(-\frac{t}{\tau_m}\right) \quad (j = 1, 2)$$

Introducing the potential functions Φ and Ψ, we can reduce the equations of motion to a system of integro-differential equations (a and b are the velocities or propagation of the longitudinal and transverse elastic waves

$$L_0(\Delta\Phi) = \frac{1}{a^2}\frac{\partial^2\Phi}{\partial t^2}, \quad M_0(\Delta\Psi) = \frac{1}{b^2}\frac{\partial^2\Psi}{\partial t^2} \tag{1.2}$$

$$L_0(\zeta) = \zeta(t) - \int_0^t [\alpha_0 f_1(t-\xi) + 2\beta_0 f_2(t-\xi)]\,\zeta(\xi)\,d\xi$$

$$M_0(\zeta) = \mu^{-1}M(\zeta), \quad \alpha_0 = \frac{\lambda}{\lambda+2\mu}, \quad \beta_0 = \frac{\mu}{\lambda+2\mu}$$

$$\left(u = \frac{\partial\Phi}{\partial x} + \frac{\partial\Psi}{\partial y},\ v = \frac{\partial\Phi}{\partial y} - \frac{\partial\Psi}{\partial x}\right)$$

Let us consider the problem of diffraction of an arbitrary visco-elastic wave on the contour C (Fig. 1). We have two basic types of boundary conditions at the contour

$$u_n = v_s = 0 \quad (C) \tag{1.3}$$

$$u_n = \sigma_{ns} = 0 \quad (C) \tag{1.4}$$

corresponding to the case of a contour rigidly bound to the surrounding medium, and to the case when there is no friction between the medium and the contour. Here n and s denote the normal to and the arc of the contour C.

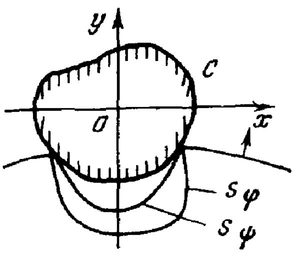

Fig. 1

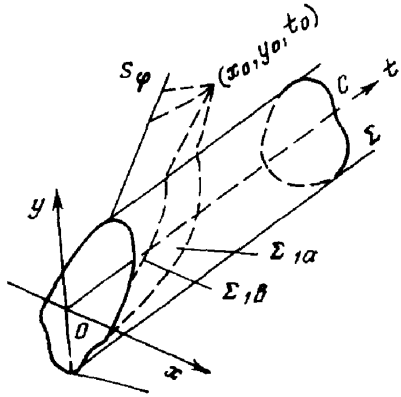

Fig. 2

The following conditions hold at the fronts S_φ and S_ψ of the reflected longitudinal and transverse waves

$$\Phi = \Phi_0 \quad (S_\varphi), \quad \Psi = \Psi_0 \quad (S_\psi) \tag{1.5}$$

where Φ_0 and Ψ_0 are the potentials of the incident visco-elastic wave. In what follows, we shall assume that $\Psi_0 = 0$. Let us set

$$\Phi = \varphi + \Phi_0, \quad \Psi = \psi$$

where φ and ψ are the potentials of the diffracted waves. The potentials φ and ψ can be found from the integro-differential equation

$$L_0(\Delta\varphi) = \frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}, \quad M_0(\Delta\psi) = \frac{1}{b^2}\frac{\partial^2\psi}{\partial t^2} \tag{1.6}$$

with the boundary and initial conditions

$$\frac{\partial\varphi}{\partial n} = -\frac{\partial\Phi_0}{\partial n} + \frac{\partial\psi}{\partial s}, \quad \frac{\partial\psi}{\partial n} = -\frac{\partial\Phi_0}{\partial s} - \frac{\partial\varphi}{\partial s} \quad (C) \tag{1.7}$$

or

$$\frac{\partial\varphi}{\partial n} = -\frac{\partial\Phi_0}{\partial n}, \quad \psi = 0 \quad (C) \tag{1.8}$$

$$\varphi = 0 \quad (S_\varphi), \quad \psi = 0 \quad (S_\psi)$$

$$\varphi = \frac{\partial\varphi}{\partial t} = \psi = \frac{\partial\psi}{\partial t} = 0 \quad (t \leqslant 0)$$

Let us consider the process of diffraction of a longitudinal visco-elastic wave in the space (x, y, t) using the methods of [5, 6]. As in [3], the approximate solution of the problem (1.6)—(1.8) in the (x, y, t)-space has the form

$$\varphi(x_0, y_0, t_0) = \frac{1}{2\pi} L_{01}\left\{\frac{\partial}{\partial t_0}\iint\limits_{\Sigma_{1a}} L_0\left(\varphi\frac{\partial v_a}{\partial n} - v_a\frac{\partial \varphi}{\partial n}\right)dq\right\} \tag{1.9}$$

$$\psi(x_0, y_0, t_0) = \frac{1}{2\pi} M_{01}\left\{\frac{\partial}{\partial t_0}\iint\limits_{\Sigma_{1b}} M_0\left(\psi\frac{\partial v_b}{\partial n} - v_b\frac{\partial \psi}{\partial n}\right)dq\right\}$$

$$L_{01}(\zeta) = \zeta + \frac{c_{12}}{2}\int_0^t \zeta(\xi)\,d\xi, \quad c_{12} = \int_0^\infty \alpha h(\alpha)\,d\alpha + \sum_{m=1}^{m_0} \gamma_m \tau_m^{-1}$$

$$M_{01}(\zeta) = \zeta + \frac{c_{22}}{2}\int_0^t \zeta(\xi)\,d\xi, \quad c_{22} = \int_0^\infty \alpha h_2(\alpha)\,d\alpha + \sum_{m=1}^{m_0} \gamma_{m2} \tau_m^{-1}$$

$$h(\alpha) = \alpha_0 h_1(\alpha) + 2\beta_0 h_2(\alpha), \quad \gamma_m = \alpha_0\gamma_{m1} + 2\beta_0\gamma_{m2}$$

where Σ_{1a} and Σ_{1b} denote the parts of the cylindrical surface Σ (Fig. 2) in the (x, y, t)-space intercepted by the cones of influence $a^2(t_0 - t)^2 - r^2 = 0$, $b^2(t_0 - t)^2 - r^2 = 0$, $r^2 = (x_0 - x)^2 + (y_0 - y)^2$ from an arbitrary point (x_0, y_0, t_0), while v_a and v_b represent the fundamental solutions of the integro-differential equations (1.6)

$$v_c(x, y, t; x_0, y_0, t_0) = \int_0^{V_c} \Phi_c(t_0 - t, r\operatorname{ch}\alpha)\,d\alpha \quad (c = a, b)$$

$$\Phi_c(t, \xi) = L^{(0)}\left\{\frac{\sqrt{1 - f_c(p)}}{cp}\exp\left[-\frac{p\xi c}{\sqrt{1 - f_c(p)}}\right]\right\}$$

where $L^{(0)}(\zeta)$ is the inverse Laplace transform, $f_a(p)$ and $f_b(p)$ are Laplace transforms of the functions $\alpha_0 f_1(t) + 2\beta_0 f_2(t)$ and $f_2(t)$, and V_c is the Volterra function of the corresponding wave equation [5].

For a Maxwellian body, in particular, we have

$$v_c(x, y, t; x_0, y_0, t_0) = \exp\left(-\frac{t_0 - t}{2\tau}\right)\int_0^{V_c} I_0 \times \left[\frac{1}{2c\tau}\sqrt{c^2(t_0 - t)^2 - r^2\operatorname{ch}^2\alpha}\right]d\alpha$$

where τ denotes the relaxation time. When the bodies are elastic and $f_1(t) = f_2(t) = 0$ functions v_a and v_b become the Volterra function.

When friction is absent between the contour C and the medium, the potential $\psi \equiv 0$ and only the longitudinal wave is reflected.

Since the values of the functions φ and ψ on the contour C are not known, it follows that a limiting passage of the point (x_0, y_0, t_0) to the surface Σ yields a system of integral equations for determining φ and ψ on Σ. Solving these equations for φ and ψ on Σ followed by substitution into (1.9), yields a solution for the diffraction problem. Thus the general problem of diffraction of a visco-elastic wave can be reduced to that of obtaining a solution of the type (1.9) of the Volterra equations for φ and ψ on Σ.

We shall consider those specific diffraction problems, which can be solved in quadratures using the general formulas (1.9).

2. **Diffraction of a visco-elastic wave on a semi-infinite cut.** Before solving the diffraction problem, we shall consider a reflection of a visco-elastic cylindrical wave from an infinite wall. We direct the x-axis along the wall, and the y-axis normally to it (Fig.3), and replace n in (1.9) by y.

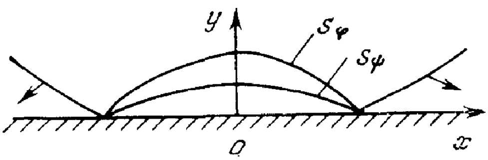

Fig.3

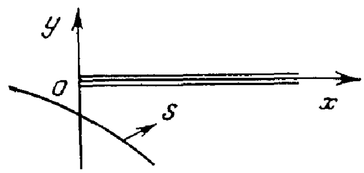

Fig. 4

The surface Σ in the (x, y, t)-space is represented in this case by the half-plane $t \geqslant 0, -\infty < x < \infty$ the front edge of which is bounded by the curve

$$f(x, 0, t) = 0 \tag{2.1}$$

where $f(x, y, t) = 0$ denotes the front of the incident longitudinal visco-elastic wave.

Consider the point $(x_0, -y_0, t_0)$ and apply the formulas (1.9) at $y > 0$. Since this point lies outside the region $y > 0$, the formulas (1.9) yield

$$0 = \frac{1}{2\pi} L_{01} \left\{ \frac{\partial}{\partial t_0} \iint\limits_{\Sigma_{1a}} L_0 \left(\varphi \frac{\partial v_{a1}}{\partial y} - v_{a1} \frac{\partial \varphi}{\partial y} \right) dq \right\} \tag{2.2}$$

$$0 = \frac{1}{2\pi} M_{01} \left\{ \frac{\partial}{\partial t_0} \iint\limits_{\Sigma_{1b}} M_0 \left(\psi \frac{\partial v_{b1}}{\partial y} - v_{b1} \frac{\partial \psi}{\partial y} \right) dq \right\}$$

$$v_{c1}(x, y, t; x_0, y_0, t_0) = v_c(x, y, t; x_0, -y_0, t_0)$$

It is evident that when $y = 0$, the functions v_{c1} and v_c satisfy the conditions

$$v_{c1} = v_c, \quad \partial v_{c1}/\partial y = -\partial v_c/\partial y \quad (y = 0)$$

Consequently when $y = 0$ and $\partial\varphi/\partial y$ and $\partial\psi/\partial y$ are specified, then combining the right-and left-hand sides of the formulas (1.9) $(n = y)$ and (2.2) yields

$$\varphi(x_0, y_0, t_0) = -\frac{1}{\pi} L_{01} \left\{ \frac{\partial}{\partial t_0} \iint\limits_{\Sigma_{1a}} v_a \frac{\partial L_0(\varphi)}{\partial y} dq \right\} \tag{2.3}$$

$$\psi(x_0, y_0, t_0) = -\frac{1}{\pi} M_{01} \left\{ \frac{\partial}{\partial t_0} \iint\limits_{\Sigma_{1b}} v_b \frac{\partial M_0(\psi)}{\partial y} dq \right\}$$

Similarly, by specifying φ and ψ on Σ we obtain

$$\varphi(x_0, y_0, t_0) = \frac{1}{\pi} L_{01} \left\{ \frac{\partial}{\partial t_0} \iint\limits_{\Sigma_{1a}} \varphi \frac{\partial L_0(v_a)}{\partial y} dq \right\} \tag{2.4}$$

$$\psi(x_0, y_0, t_0) = \frac{1}{\pi} M_{01} \left\{ \frac{\partial}{\partial t_0} \iint\limits_{\Sigma_{1b}} \psi \frac{\partial M_0(v_b)}{\partial y} dq \right\}$$

In particular, if the friction at the wall is zero, the potential $\psi \equiv 0$ and the first formula of (2.3) yields an exact solution of the problem in quadratures without the need to reduce it to a system of integral equations in φ and ψ on Σ.

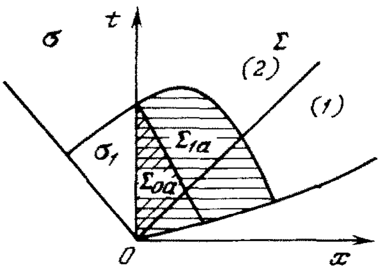

Fig. 5

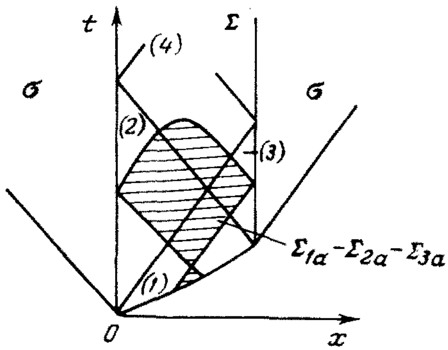

Fig. 6

Let us now conctruct the solution of the diffraction problem on a semi-infinite cut. Let a longitudinal wave defined by the potential $\Phi_0(x, y, t)$ (Fig. 4) fall on the semi-infinite cut $y = 0, 0 \leqslant x < \infty$. The surface Σ in the (x, y, t)-space is represented by a quarter of the plane, (Fig. 5) the front edge of which is bounded by the curve (2.1). We assume that the cut is rigid and the friction between the cut and the medium is zero. Then $\psi \equiv 0$ and the problem reduces to that of determining the potential φ satisfying Eq. (1.6) together with the boundary and initial conditions

$$\frac{\partial \varphi}{\partial y} = -\frac{\partial \Phi_0}{\partial y} \quad (y = 0, 0 \leqslant x < \infty), \quad \varphi = \frac{\partial \varphi}{\partial t} = 0 \quad (t \leqslant 0) \tag{2.5}$$

Let us divide the surface Σ into two parts, (1) and (2), as shown in Fig. 5. When the potential φ is acted upon by the points of the region (1), we can use for φ the first formula of (2.3), i.e.

$$\varphi(x_0, y_0, t_0) = \frac{1}{\pi} L_{01} \left\{ \frac{\partial}{\partial t_0} \iint\limits_{\Sigma_{1a}} v_a \frac{\partial L_0(\Phi_0)}{\partial y} dq \right\} \tag{2.6}$$

Similarly, under the action of both regions, (1.) and (2), we have for φ

$$\varphi(x_0, y_0, t_0) = -\frac{1}{\pi} L_{01} \left\{ \frac{\partial}{\partial t_0} \iint\limits_{\Sigma_{1a}+\sigma_1} v_a \frac{\partial L_0(\varphi)}{\partial y} dq \right\} \tag{2.7}$$

where σ_1 is the part of σ in the plane $y=0$ lying outside Σ (see Fig. 5) on which the quantity $\partial\varphi/\partial y$ is not known.

When $y=0$, we find by virtue of the first condition of (2.5) that φ is an odd function of the coordinate y, therefore $\varphi=0$ on σ.

To find $\partial\varphi/\partial y$ at the points of σ we perform a limiting passage of the point (x_0, y_0, t_0) to the surface σ. We have

$$0=\frac{1}{\pi}L_{01}\left\{\frac{\partial}{\partial t_0}\iint\limits_{\Sigma_{11}+\sigma_{10}} v_a\frac{\partial L_0(\varphi)}{\partial y}\,dq\right\}$$

or

$$0=\iint\limits_{\Sigma_{11}+\sigma_{10}}\frac{\partial v_a}{\partial t_0}\,\frac{\partial L_0(\varphi)}{\partial y}\,dq \tag{2.8}$$

Changing the order of integration in the inner integrals of the right-hand side of (2.8), we reduce it to the form

$$\iint\limits_{\Sigma_{11}+\sigma_{10}}\frac{\partial\varphi}{\partial y}\,\frac{Q_a(t_0,t;x_0,x)}{\sqrt{a^2(t_0-t)^2-(x_0-x)^2}}\,dq=0 \tag{2.9}$$

$$Q_a(t_0,t;x_0,x)=[1-f_0(0)+f_0(t_0-t-|x_0-x|)]\times$$

$$\left\{\Phi_a(t_0-t,t_0-t)+\sqrt{a^2(t_0-t)^2-(x_0-x)^2}\int\limits_0^{V_a}\frac{\partial\Phi_a}{\partial t_0}\,d\alpha\right\}$$

$$f_0(t_0-t-|x_0-x|)-f_0(0)=\int\limits_t^{t_0-|x_0-x|}[\alpha_0 f_1(t-\xi)+2\beta_0 f_2(t-\xi)]\,d\xi$$

where due notice has been taken of the fact that the quantity $\partial v_a/\partial t_0$ is equal to the ratio of the term within the curly brackets in the expressions for Q_a, to

$$\sqrt{a^2(t_0-t)^2-(x_0-x)^2}$$

Using the intrinsic coordinates

$$\mu=at+x,\quad \nu=at-x \tag{2.10}$$

we write the formula (2.9) in the form

$$\int\limits_0^{\mu_0}\frac{d\mu}{\sqrt{\mu_0-\mu}}\left\{-\int\limits_{A(\mu)}^{\mu}\frac{\partial\Phi_0}{\partial y}\,\frac{Q_{0a}(\mu_0,\mu;\nu_0,\nu)}{\sqrt{\nu_0-\nu}}+\int\limits_{\mu}^{\nu_0}z\,\frac{Q_{0a}(\mu_0,\mu;\nu_0,\nu)}{\sqrt{\nu_0-\nu}}\,d\nu\right\}=0 \tag{2.11}$$

$$Q_{0a}(\mu_0,\mu;\nu_0,\nu)=Q_a(t_0,t;x_0,x)$$

where $z=\partial\varphi/\partial y$ on σ, and $\nu=A(\mu)$ is the equation of the front edge of Σ in the coordinates (2.10).

It is clear that (2.11) is a double integral Abel equation in z. Solving this equation we obtain

$$z=\frac{1}{\pi}\,\frac{\partial}{\partial\nu_0}\int\limits_{\mu_0}^{\nu_0}\frac{d\nu}{\sqrt{\nu_0-\nu}}\int\limits_{A(\mu_0)}^{\mu_0}\frac{\partial\Phi_0}{\partial y}\,\frac{Q_{0a}(\mu_0,\mu_0;\nu,\xi)}{\sqrt{\nu-\xi}}\,d\xi \tag{2.12}$$

$$Q_{0a}(\mu_0,\mu_0;\nu_0,\nu_0)\equiv 1$$

Substituting the value of z into the right-hand side of the formula (2.7) and performing the necessary transformation, we obtain the final expression for φ under the action of the points of the regions (1) and (2)

$$\varphi(x_0, y_0, t_0) = \frac{1}{\pi} L_{01}\left\{\frac{\partial}{\partial t_0} \iint\limits_{\Sigma_{1a}-\Sigma_{0a}} v_a \frac{\partial L_0(\Phi_0)}{\partial y} dq\right\} \tag{2.13}$$

The region $\Sigma_{1a} - \Sigma_{0a}$ shown in Fig. 5 is called the equivalent domain of integration. Formulas (2.6) and (2.13) yield the solution of the diffraction problem in quadratures, and (2.13) determines the potential φ with the reflected and diffracted waves both taken into account.

Example. Let the incident wave be plane and generated by the pressure $F(t - t_1)$ applied at the points of the straight line

$$x \sin\gamma + y\cos\gamma + \xi_0 = 0 \quad (\gamma > 0,\ t_1 < 0)$$

where ξ_0 is the distance between the straight line and the front point of the cut.

Using the solution of the problem and assuming that the medium complies with the Maxwell's model, we obtain the following expression for the stresses σ_{ij} in the reflected wave defined by (2.6):

$$\sigma_{xx} = \sin^2\gamma \sum_{j=1}^{2} Q_j(x, y, t), \quad \sigma_{yy} = \cos^2\gamma \sum_{j=1}^{2} Q_j(x, y, t) \tag{2.14}$$

$$\sigma_{xy} = \sin 2\gamma \sum_{j=1}^{2} (-1)^{1+j} Q_j(x, y, t), \quad x_{1,2} = x\sin\gamma \pm y\cos\gamma$$

$$Q_j(x, y, t) = -\exp\left(-\frac{x_j}{2a\tau}\right) F\left(t - \frac{x_j}{a}\right) H\left(t - \frac{x_j}{a}\right) +$$

$$\frac{x_j}{2a\tau}\int\limits_{x_j/a}^{t} F(t-\xi)\exp\left(-\frac{\xi}{2\tau}\right)\left(\xi^2 - \frac{x_j^2}{a^2}\right)^{-1/2} I_1\left[\frac{1}{2\tau}\sqrt{\xi^2 - \frac{x_j^2}{a^2}}\right] d\xi$$

The formula (2.14) contains the relaxation time τ which determines the decay of stress with time and distance.

3. Diffraction of a visco-elastic wave on a cut of finite length. Let us extend the result of Sect. 2 to the case of a cut of width l. In this case the half strip of width l in the (x, y, t)-space (see Fig. 6) on which the boundary condition (2.5) has the form

$$\partial\varphi/\partial y = -\partial\Phi_0/\partial y \quad (y = 0, 0 \leqslant x \leqslant l)$$

will represent the surface Σ.

Let us divide the surface Σ, as shown in Fig. 6, into the regions (1) – (3). When the potential φ is under the influence of the points of the region (1) or of (1) and (2), we obtain φ from the formulas (2.6) and (2.13). When the points of (1) and (3) exert their influence, we also have

$$\varphi(x_0, y_0, t_0) = \frac{1}{\pi} L_{01}\left\{\frac{\partial}{\partial t_0} \iint\limits_{\Sigma_{1a}-\Sigma_{2a}} v_a \frac{\partial L_0(\Phi_0)}{\partial y} dq\right\}$$

where the region $\Sigma_{1a} - \Sigma_{2a}$ takes into account the diffracted wave from the right-hand end of the cut.

We determine φ in the same manner when the points of the regions (1) – (4) exert their influence, and we have

$$\varphi(x_0, y_0, t_0) = \frac{1}{\pi} L_{01} \left\{ \frac{\partial}{\partial t_0} \iint\limits_{\Sigma_{1a}-\Sigma_{2a}-\Sigma_{3a}} v_a \frac{\partial L_0(\Phi_0)}{\partial y} \, dq \right\}$$

We obtain φ when the subsequent diffracted waves are taken into account, in the same manner. Using the known values of the potential φ, we can now determine the components of the displacement vector and stress tensor.

We can pose another problem related to the problem in question.

Let a transverse shear wave in which the displacement is perpendicular to the (x, y)-plane fall on a cut of finite length. Also in this case only the shear wave will be reflected.

The problem can be reduced to that of determining the displacement W directed perpendicularly to the (x, y)-plane. The displacement satisfies the second equation of (1.2) as written for the transverse potential ψ, and the boundary and initial conditions

$$\frac{\partial W}{\partial y} = 0 \quad (\Sigma), \quad W = W_0 \quad (S_\psi)$$

$$W = W_0, \quad \frac{\partial W}{\partial t} = \frac{\partial W_0}{\partial t} (t \leqslant 0)$$

Setting $W = \varphi + W_0$ we obtain, for the perturbed displacement of φ, a problem analogous to the problem of diffraction of a longitudinal wave on a finite cut without friction: we must only replace the operator $L_0(\zeta)$ by $M_0(\zeta)$ and the velocity a by the velocity b. Other problems of diffraction can also be solved using the methods described in Sect. 1 and 2.

N o t e. In the continuous electromagnetic media with a finite specific conductivity σ, the wave field is described by the Maxwell's equations.

For the media with finite conductivity, in particular for metals, the current density $\mathbf{j}$ is proportional to the electric field intensity $\mathbf{E}$, i.e. $\mathbf{j} = \sigma \mathbf{E}$. We can introduce for such media a vector function $\mathbf{A}$ according to the formula ($\mathbf{B}$ is the magnetic induction vector and c denotes the speed of light)

$$\mathbf{B} = \operatorname{rot} \mathbf{A}, \quad \mathbf{E} = -\frac{1}{c} \frac{\partial A}{\partial t}, \quad \operatorname{div} \mathbf{A} = 0$$

and the Maxwell's equations will then reduce to a single equation

$$\Delta \mathbf{A} = \frac{4\pi\mu\sigma}{c^2} \frac{\partial \mathbf{A}}{\partial t} + \frac{\varepsilon\mu}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} \tag{3.1}$$

where ε and μ are the dielectric constant and magnetic permeability of the medium.

Equation (3.1) is equivalent to the equation of propagation of longitudinal or transverse waves through a visco-elastic body, the material of which complies with the Maxwell's model. The term containing $\partial A / \partial t$ leads to the appearance in the solution of a decaying multiplier, and the latter will depend, in general, not only on time but also on the points belonging to the space. The visco-elastic media experience the same phenomena.

Equation (3.1) can be reduced to the form

$$L_1(\Delta A) = \frac{\varepsilon\mu}{c^2}\frac{\partial^2 A}{\partial t^2}$$

$$L_1(\zeta) = \zeta(t) - \int_0^t \frac{1}{\tau}\exp\left(-\frac{t-\xi}{\tau}\right)\zeta(\xi)\,d\xi, \quad \tau = \frac{4\pi\sigma}{\varepsilon}$$

and the quantity τ is equivalent to the relaxation time in the visco-elastic Maxwellian body.

Thus, since the equations describing the process of propagation of the visco-elastic and electromagnetic waves through the media with finite conductivity are equivalent, it follows that many results concerning the wave propagation and diffraction problems in visco-elastic media can be transposed to the corresponding problems in electro-magnetic media.

REFERENCES

1. Guz', A.N. and Golovchan, V.T., Diffraction of elastic waves in multi-connected bodies. Kiev, "Naukova Dumka", 1972.

2. Sagomonian, A.Ia. Penetration. Moscow State University, 1974.

3. Smirnov, V.I. and Sobolev, S.L., Novel method of solving the plane problem of elastic oscillations. Tr. Seismolog. Inst. Akad. Nauk SSSR, No. 20, 1932.

4. Filippov, A.F., Some problems of diffraction of plane elastic waves. PMM, Vol. 20, No. 6, 1956.

5. Filippov, I.G., On the theory of diffraction of cylindrical elastic waves and weak shock waves. PMM, Vol. 28, No. 2, 1964.

6. Filippov, I.G. and Cheban, V.G., Unsteady motions of continuous compressible media. Kishinev, Shtiintsa, 1973.

7. Fridman, M.M., Diffraction of a plane elastic wave on a stress-free, semi-infinite rectilinear cut. Dokl. Akad. Nauk SSSR, Vol. 66, No. 1, 1949.

Translated by L.K.